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THE BELL SYSTEM TECHNICAL JOURNAL
Vol. 53, No. 4, April 1974
Printed in U.S.A.

Coupling of Nearly Degenerate Modes in Parallel Asymmetric Dielectric Waveguides

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(Manuscript received May 15, 1973)

The coupling of modes in two parallel dielectric waveguides is studied. The individual waveguides are assumed to be asymmetric and unlike each other. If the individual waveguides support modes with nearly equal propagation constants β_2 *and* $\beta_4 = \beta_2 + 2\Delta$*, then the double waveguide system will support two new modes with propagation constants* $\beta_- = \beta_2 - \bar{\delta}$ *and* $\beta_+ = \beta_4 + \bar{\delta}$*. The shift* $\bar{\delta}$ *is related to* Δ *and to the shift* δ *which would occur if the original modes were degenerate;* δ *is expressed in terms of the parameters describing the asymmetric double waveguide system. The field distributions of the new modes are approximately even and odd combinations of those of the original modes in the isolated waveguides; the relative amplitudes with which they are combined depend upon the amount of mismatching* Δ*. As the modes travel down the waveguide system, they partially cancel and add, thus transferring power. A power transfer ratio* F *is defined and is shown to decrease rapidly as* Δ/δ *increases. The beat length* L *depends upon both* δ *and* Δ/δ*; it also decreases as* Δ/δ *increases. A numerical example is given to illustrate the effects of mismatching and to demonstrate the feasibility of constructing a mode-coupling device. Possibilities of tuning the device to reduce mismatching are discussed.*

I. INTRODUCTION

Coupling of degenerate modes of parallel optical waveguides has been discussed by Kapany[1] and, to a greater extent, by Marcuse.[2] Such coupling is of particular interest in the field of fiber optics, since it may cause undesirable crosstalk between adjacent optical fibers used for light transmission. Marcuse[2] has applied the theory of degenerate mode coupling to the problem of crosstalk between cladded optical fibers embedded in a lossy medium and between cladded dielectric slab waveguides. The fabrication of devices which would actually take advantage of mode coupling, such as for light switching, modulation, or power transferral,[3] is fraught with practical difficulties, since the specification of physical parameters must necessarily be stringent. These difficulties require us to view the theory of optical waveguide coupling from a new vantage point.

Let us first sketch briefly what is known. If two optical waveguides each have a mode with the same propagation constant β, then when the two waveguides are placed parallel to each other, the double waveguide system supports two new modes whose propagation constants are $\beta_+ = \beta + \delta$ and $\beta_- = \beta - \delta$. These two modes are approximately symmetric and anti-symmetric combinations of the original modes in the isolated waveguides. The shift in propagation constant, δ, is related to the coupling coefficients involved in a description of the modes by means of general coupled line equations. It can also be expressed via a perturbation treatment of Maxwell's equations. Since the superimposed modes travel down the double waveguide system at different phase velocities, they alternately add and cancel. If the waveguides are lossless, power is transferred back and forth over a beat length $L = \pi/(2\delta)$. On the other hand, Marcuse shows that, if the waveguides are lossy, they tend to equalize the power they carry, provided the modes travel far enough. A lossy external medium also causes mode loss. Marcuse further states that only degenerate modes exchange a significant amount of power if their coupling mechanism is independent of length.

With this abbreviated version of the present theory in mind, we see several criteria which a mode coupling device should satisfy: (*i*) the core and cladding of each waveguide should be lossless, (*ii*) the medium external to the waveguides should be lossless, and (*iii*) the two waveguides should have a degenerate mode. (There are also other criteria, such as that the waveguide walls be free from imperfections, but they are not discussed.)

The first criterion is an important one and certainly merits further study. In this paper, however, we avoid the issue by assuming that the device we fabricate has lossless waveguides. A subtler way to put this is to say that the device is short enough that losses can be ignored.

The second criterion is satisfied by assuming that the claddings of the two waveguides are contiguous and that there is no medium external to them. Instead of thinking in terms of two optical fibers, we consider two dielectric slab waveguides placed next to each other. Fabrication would be similar to that currently used in the production of double heterostructure lasers and modulators.[4,5] Each waveguide will consist of a slab of high refractive index surrounded by two slabs of lower index. Since the double waveguide device will have a central slab common to both waveguides, the device can be modelled by a 5-dielectric-slab model.

The third criterion, that the two waveguides have a degenerate mode, motivates our present study. In practice, it is very difficult to fabricate a device with degenerate modes. It is therefore quite important to know how well the device will operate if the propagation constants for the modes are slightly mismatched. We study the effect of mismatching on the beat length and on the capability of the device to transfer power. We also discuss methods of tuning the device after it is fabricated. The tuning could be used to match the propagation constants more closely. It might also be used dynamically, thus offering the possibility of utilizing the double waveguide system as a light switch or a modulator.

II. FORMULATION

We adopt the standard slab model of an absorptionless dielectric medium. The optical dielectric $K(x)$, i.e., the square of the refractive index, is assumed to vary only with x and to take the piecewise constant form shown in Fig. 1. If the waves are assumed to travel in the

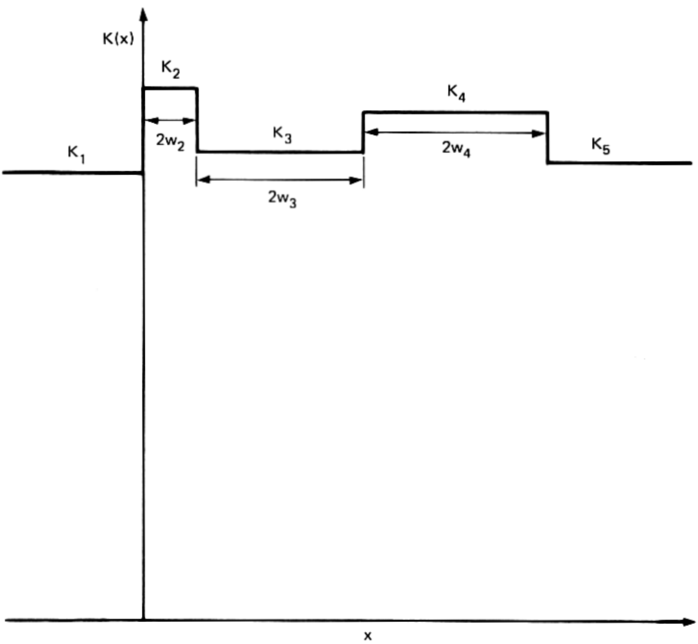

Fig. 1—The optical dielectric profile $K(x)$ for the five-slab model.

z-direction with propagation constant β, then the electric and magnetic fields are independent of y and can be expressed as

$$E = e(x) \exp i(\omega t - \beta z),$$
$$H = h(x) \exp i(\omega t - \beta z),$$

where ω is the angular frequency of the light and t is the time. Both TE and TM modes exist. It follows from Maxwell's equations that the electric field $e_y(x)$ of a TE mode is described by

$$\frac{d^2 e_y}{dx^2} + [k^2 K(x) - \beta^2] e_y = 0, \tag{1}$$

and that the magnetic field $h_y(x)$ of a TM mode is described by

$$K(x) \frac{d}{dx}\left(\frac{1}{K(x)} \frac{dh_y}{dx}\right) + [k^2 K(x) + \beta^2] h_y = 0. \tag{2}$$

It is required that

$$e_y, \frac{de_y}{dx},$$
$$h_y, \frac{1}{K(x)} \frac{dh_y}{dx}$$

be continuous. Since $K(x)$ is piecewise constant, solution of eqs. (1) and (2) subject to the above conditions is straightforward. The solution of (1) is

$$\begin{aligned}
e_y(x) &= A \exp p_1 x \qquad x < 0 \\
&= A[(p_1/p_2) \sin p_2 x + \cos p_2 x] \qquad 0 < x < 2w_2 \\
&= AC_2[1 + (p_1/p_2) T_2][-X \sinh p_3(x - 2w_2) \\
&\qquad + \cosh p_3(x - 2w_2)] \qquad 2w_2 < x < 2(w_2 + w_3) \\
&= AC_2C_3[1 + (p_1/p_2)T_2][1 - XT_3] \\
&\quad \times [(p_3/p_4) Y \sin p_4(x - 2w_2 - 2w_3) + \cos p_4(x - 2w_2 - 2w_3)] \\
&\qquad 2(w_2 + w_3) < x < 2(w_2 + w_3 + w_4) \\
&= AC_2C_3C_4[1 + (p_1/p_2)T_2][1 - XT_3] \\
&\quad \times [1 + (p_3/p_4) YT_4] \exp p_5(2w_2 + 2w_3 + 2w_4 - x) \\
&\qquad 2(w_2 + w_3 + w_4) < x,
\end{aligned} \tag{3}$$

with

$$\begin{aligned}
p_i(\beta) &= (\beta^2 - k^2 K_i)^{\frac{1}{2}} \qquad i = 1, 3, 5, \\
p_i(\beta) &= (k^2 K_i - \beta^2)^{\frac{1}{2}} \qquad i = 2, 4,
\end{aligned} \tag{4}$$

$$\begin{aligned}
C_2(\beta) &= \cos 2p_2 w_2, \qquad & T_2(\beta) &= \tan 2p_2 w_2, \\
C_3(\beta) &= \cosh 2p_3 w_3, \qquad & T_3(\beta) &= \tanh 2p_3 w_3, \\
C_4(\beta) &= \cos 2p_4 w_4, \qquad & T_4(\beta) &= \tan 2p_4 w_4,
\end{aligned} \tag{5}$$

$$X(\beta) = \frac{1}{p_3}\left[\frac{p_2T_2 - p_1}{1 + (p_1/p_2)T_2}\right], \tag{6}$$

$$Y(\beta) = \frac{1}{p_3}\left[\frac{p_4T_4 - p_5}{1 + (p_5/p_4)T_4}\right]. \tag{7}$$

The amplitude A is arbitrary. Equation (3) satisfies the continuity condition on $e_y(x)$ everywhere, and that on de_y/dx at all but the point $x = 2(w_2 + w_3)$. The continuity condition at this point leads to the eigenvalue equation

$$T_3(\beta) = \frac{X(\beta) + Y(\beta)}{1 + X(\beta)Y(\beta)}, \tag{8}$$

which determines the values of the propagation constant β for which discrete modes can exist.

For TM modes, the analogous equations are formed by replacing p_i by $\tilde{p}_i$ and w_i by $\tilde{w}_i$, where

$$\begin{aligned} K_i\tilde{p}_i(\tilde{\beta}) &= (\tilde{\beta}^2 - k^2K_i)^{\frac{1}{2}} && i = 1, 3, 5, \\ K_i\tilde{p}_i(\tilde{\beta}) &= (k^2K_i - \tilde{\beta}^2)^{\frac{1}{2}} && i = 2, 4, \\ \tilde{w}_i &= K_iw_i && i = 2, 3, 4, \end{aligned}$$

and $\tilde{\beta}$ denotes the propagation constant for a TM mode. For simplicity of exposition, we have mainly confined our analysis to that of TE modes. It should be clear how to do the corresponding analysis for TM modes.

We remark that the above analysis is quite general and makes no assumptions about the relative heights or widths involved in the dielectric profile $K(x)$ sketched in Fig. 1. By making appropriate choices of the parameters, we could deduce from eqs. (3) to (8) the corresponding equations for a single asymmetric or symmetric waveguide, for example. Two cases which interest us particularly are: (*i*) $K_3 = K_4 = K_5$ with $K_2 > K_1$ and $K_2 > K_3$, and (*ii*) $K_1 = K_2 = K_3$ with $K_4 > K_3$ and $K_4 > K_5$. Each of these models an isolated waveguide. We call the first of these (with high dielectric region K_2) guide II and the other (with high dielectric region K_4) guide IV. The eigenvalue equation (8) then reduces to

$$\begin{aligned} X(\beta) &= 1 && \text{guide II}, \\ Y(\beta) &= 1 && \text{guide IV}. \end{aligned}$$

These may appear more familiar to the reader when cast in the standard form for a single asymmetric guide[6]

$$\tan 2w_2p_2 = \frac{(p_1/p_2) + (p_3/p_2)}{1 - (p_1/p_2)(p_3/p_2)} \qquad \text{guide II,} \tag{9}$$

$$\tan 2w_4p_4 = \frac{(p_3/p_4) + (p_5/p_4)}{1 - (p_3/p_4)(p_5/p_4)} \qquad \text{guide IV.} \tag{10}$$

For the model of two adjacent waveguides, we place guides II and IV adjacent to each other as illustrated in Fig. 1, with $K_2 > \max\{K_1, K_3\}$, $K_4 > \max\{K_3, K_5\}$, and $w_i > 0$ $(i = 1, 2, 3, 4)$.

It is also a relatively straightforward procedure to write down precisely how many modes can exist with a given dielectric profile $K(x)$.[7] Although we omit such expressions here, we do comment that, just as a single asymmetric guide may not be able to support a propagating mode, so also an asymmetric double waveguide structure is not always capable of mode propagation. If $K_1 = K_3 = K_5$, though, so that the structure is composed of two parallel symmetric (but not necessarily identical) waveguides, then there is always at least one mode.

III. NEARLY DEGENERATE MODES

Let β_2 and β_4 denote solutions of $X(\beta) = 1$ and $Y(\beta) = 1$, respectively; β_2 and β_4, then, are propagation constants for modes in guides II and IV if the guides were isolated from each other. We need make no assumption about the order of each mode. In practice, though, both propagation constants are likely to be associated with zeroth order modes. For definiteness in notation, we assume that $\beta_4 \geqq \beta_2$ and write*

$$\beta_4 - \beta_2 = 2\Delta.$$

We now assume that Δ is "small," i.e., that the two modes are nearly degenerate. This assumption, which is fundamental to the remainder of the analysis, is stated more explicitly later [eq. (18)]. Frequently, it does not matter (to the order of approximation used) whether β_2 or β_4 is used in the evaluation of an expression. In such instances, it sometimes is helpful to use the notation $\beta_0 (\doteq \beta_2 \doteq \beta_4)$.

Our task is now to determine values of β which satisfy (8). Our experience with the degenerate case $(\Delta = 0)$ leads us to expect that

* In the numerical example of Section VI, we relax this notation to read $|\beta_4 - \beta_2| = 2\Delta$, where it is not known *a priori* whether β_2 or β_4 is larger. This should not be confusing when taken in context.

there will be two solutions β_+ and β_- close to β_0. A study of the coupled line equations[8] for the two modes would demonstrate that $\beta_+ = \beta_4 + \bar{\delta}$ and $\beta_- = \beta_2 - \bar{\delta}$, where $\bar{\delta}$ is expressed in terms of the (unknown) coupling coefficients.[6] We prefer to attack (8) directly; we shall verify the expressions for β_+ and β_-, prove that $\bar{\delta} > 0$, and give an explicit formula for $\bar{\delta}$.

We first show that, if β_2 and β_4 are close enough together, then (8) has no solution $\bar{\beta}$ such that $\beta_2 \leqq \bar{\beta} \leqq \beta_4$. Since $X(\beta_2) = 1$ and $X'(\beta) \leqq 0$ for all β, we know that if $(\bar{\beta} - \beta_2)/\beta_2 \ll 1$, then $0 < X(\bar{\beta}) \leqq 1$. Similarly, $Y(\bar{\beta}) \geqq 1$. Thus,

$$\frac{X(\bar{\beta}) + Y(\bar{\beta})}{1 + X(\bar{\beta})Y(\bar{\beta})} = \frac{1 + X(\bar{\beta})Y^{-1}(\bar{\beta})}{X(\bar{\beta}) + Y^{-1}(\bar{\beta})} \geqq 1.$$

But $T_3(\beta) = \tanh 2w_3p_3 < 1$ for all β, so (8) is not satisfied.

Next, suppose (8) has a solution $\beta_+ = \beta_4 + \bar{\delta}$, with $\bar{\delta} > 0$. Then if $\bar{\delta}$ is small, we know that $0 < X(\beta_+) < 1$ and $0 < Y(\beta_+) < 1$, so (8) may be written as

$$\begin{aligned} T(\beta) \equiv \tanh w_3p_3 &= \tanh\left[\tfrac{1}{2}(\tanh^{-1} X + \tanh^{-1} Y)\right] \\ &= \frac{X[1 + (1 - Y^2)^{\frac{1}{2}}] + Y[1 + (1 - X^2)^{\frac{1}{2}}]}{XY + [1 + (1 - X^2)^{\frac{1}{2}}][1 + (1 - Y^2)^{\frac{1}{2}}]}. \end{aligned} \tag{11}$$

We have

$$\begin{aligned} X(\beta_+) &= X(\beta_2 + 2\Delta + \bar{\delta}) \doteq 1 + (2\Delta + \bar{\delta})X'(\beta_0), \\ Y(\beta_+) &= Y(\beta_4 + \bar{\delta}) \doteq 1 + \bar{\delta}Y'(\beta_0). \end{aligned} \tag{12}$$

If we substitute (12) in (11) and perform a perturbation analysis under the two assumptions,

$$[\bar{\delta}(2\Delta + \bar{\delta})X'(\beta_0)Y'(\beta_0)]^{\frac{1}{2}} \ll 1, \tag{13}$$

$$[-(2\Delta + \bar{\delta})X'(\beta_0)]^{\frac{1}{2}} + [-\bar{\delta}Y'(\beta_0)]^{\frac{1}{2}} \ll 1, \tag{14}$$

we find

$$T(\beta_0) = 1 - [-(2\Delta + \bar{\delta})X'(\beta_0)]^{\frac{1}{2}}[-\bar{\delta}Y'(\beta_0)]^{\frac{1}{2}},$$

so that

$$\bar{\delta} = -\Delta + (\Delta^2 + \delta^2)^{\frac{1}{2}}, \tag{15}$$

where

$$\delta = \frac{1 - T(\beta_0)}{[X'(\beta_0)Y'(\beta_0)]^{\frac{1}{2}}}. \tag{16}$$

On the other hand, if we suppose that (8) has a solution $\beta_- = \beta_2 - \hat{\delta}$, with $\hat{\delta} > 0$, then $X(\beta_-) > 1$, $Y(\beta_-) > 1$, and (8) becomes

$$T(\beta) \equiv \tanh w_3 p_3 = \tanh \left[\tfrac{1}{2}(\tanh^{-1} X^{-1} + \tanh^{-1} Y^{-1})\right]$$
$$= \frac{X + (X^2 - 1)^{\frac{1}{2}} + Y + (Y^2 - 1)^{\frac{1}{2}}}{1 + [X + (X^2 - 1)^{\frac{1}{2}}][Y + (Y^2 - 1)^{\frac{1}{2}}]}.$$

By a procedure very much like that used to determine $\bar{\delta} = \beta_+ - \beta_4$, we find that $\hat{\delta} = \bar{\delta}$, as anticipated. Here, the roles of X' and Y' must be interchanged in (14).

The effect, then, of placing guides II and IV next to each other is to shift their (isolated) propagation constants β_2 and β_4 symmetrically outward by $\bar{\delta}$ to β_- and β_+. The physical meaning of δ in (16) is clear: it is the magnitude of the shift which would occur if guides II and IV had degenerate modes ($\Delta = 0$). We shall call δ the "degenerate shift."

Let us consider assumptions (13) and (14) in more detail. By means of (15) and (16), (13) becomes

$$1 - T(\beta_0) = 1 - \tanh w_3 p_3 \ll 1. \tag{17}$$

This, then, is essentially a restriction on the separation between the two waveguides. If they are too close, our approximations will break down. Assumption (14) and its counterpart with the roles X' and Y' reversed are, by (15), satisfied if

$$[\Delta + (\Delta^2 + \delta^2)^{\frac{1}{2}}]^{\frac{1}{2}}\{[-X'(\beta_0)]^{\frac{1}{2}} + [-Y'(\beta_0)]^{\frac{1}{2}}\} \ll 1. \tag{18}$$

This tells us how large Δ can get without invalidating the approximations.

By using eqs. (4), (6), and (7) cleverly, we see that the expressions for $X'(\beta_0)$ and $Y'(\beta_0)$ reduce to the simple forms

$$X'(\beta_0) = \frac{-\beta_0(p_2^2 + p_3^2)}{p_1 p_2^2 p_3}\,[2p_1 w_2 + 1 + (p_1/p_3)], \tag{19}$$

$$Y'(\beta_0) = \frac{-\beta_0(p_3^2 + p_4^2)}{p_3 p_4^2 p_5}\,[2p_5 w_4 + 1 + (p_5/p_3)]. \tag{20}$$

Thus, by (16),

$$\delta = \frac{p_2 p_3 p_4 [1 - \tanh w_3 p_3]}{\beta_0 \left\{ \dfrac{1}{p_1 p_5}(p_2^2 + p_3^2)(p_3^2 + p_4^2)\left[2p_1 w_2 + 1 + \dfrac{p_1}{p_3}\right]\left[2p_5 w_4 + 1 + \dfrac{p_5}{p_3}\right] \right\}^{\frac{1}{2}}}. \tag{21}$$

If we take the case of two identical symmetric waveguides ($p_1 = p_3 = p_5$, $p_2 = p_4$, $w_2 = w_4$) and use the approximation

$$\tanh w_3 p_1 \doteq 1 - 2 \exp(-2w_3 p_1),$$

then (21) reduces to

$$\delta = \frac{p_1^2 p_2^2 \exp(-2w_3 p_1)}{\beta_0 (p_1^2 + p_2^2)(1 + p_1 w_2)} \tag{22}$$

which is in agreement with results of Marcuse.[2] For TM modes, we arrive at

$$\tilde{\beta}_+ = \tilde{\beta}_4 + \tilde{\delta}, \qquad \tilde{\beta}_- = \tilde{\beta}_2 - \tilde{\delta}, \qquad \tilde{\Delta} = \tilde{\beta}_4 - \tilde{\beta}_2,$$

where

$$\tilde{\delta} = -\tilde{\Delta} + (\tilde{\Delta}^2 + \hat{\delta}^2)^{\frac{1}{2}},$$

$$\hat{\delta} = \frac{1 - \tanh \tilde{w}_3 \tilde{p}_3}{[\tilde{X}'(\tilde{\beta}_0)\tilde{Y}'(\tilde{\beta}_0)]^{\frac{1}{2}}},$$

$$\tilde{X}'(\tilde{\beta}_0) = \frac{-\tilde{\beta}_0(\tilde{p}_2^2 + \tilde{p}_3^2)}{K_1^2 K_2^2 K_3^2 \tilde{p}_1 \tilde{p}_2^2 \tilde{p}_3} \left[2K_1^2 K_3^2 \tilde{p}_1 \tilde{w}_2 + K_3^2 \left(\frac{K_1^2 \tilde{p}_1^2 + K_2^2 \tilde{p}_2^2}{\tilde{p}_1^2 + \tilde{p}_2^2} \right) + \frac{\tilde{p}_1}{\tilde{p}_3} \left(\frac{K_2^2 \tilde{p}_2^2 + K_3^2 \tilde{p}_3^2}{\tilde{p}_2^2 + \tilde{p}_3^2} \right) \right],$$

$$\tilde{Y}'(\tilde{\beta}_0) = \frac{-\tilde{\beta}_0(\tilde{p}_3^2 + \tilde{p}_4^2)}{K_3^2 K_4^2 K_5^2 \tilde{p}_3 \tilde{p}_4^2 \tilde{p}_5} \left[2K_3^2 K_5^2 \tilde{p}_5 \tilde{w}_4 + K_3^2 \left(\frac{K_4^2 \tilde{p}_4^2 + K_5^2 \tilde{p}_5^2}{\tilde{p}_4^2 + \tilde{p}_5^2} \right) + \frac{\tilde{p}_5}{\tilde{p}_3} \left(\frac{K_3^2 \tilde{p}_3^2 + K_4^2 \tilde{p}_4^2}{\tilde{p}_3^2 + \tilde{p}_4^2} \right) \right].$$

IV. A LOOK AT THE MODES

We now discuss what the modes $e_\pm(x)$ associated with $\beta_\pm$ are like in guides II and IV. The expressions for $e_\pm(x)$ are given by (3). Both the shapes of the modes and their relative amplitudes will be of interest.

We see from (3) that the *shape* of $e_\pm(x)$ in guide II is given by

$$f(\beta_\pm, x) = [(p_1/p_2) \sin p_2 x + \cos p_2 x]|_{\beta_\pm}. \tag{23}$$

Since

$$f(\beta_+, x) \doteq f(\beta_2, x) + (2\Delta + \bar{\delta}) \left.\frac{\partial f}{\partial \beta}\right|_{\beta_2},$$

$$f(\beta_-, x) \doteq f(\beta_2, x) - \bar{\delta} \left.\frac{\partial f}{\partial \beta}\right|_{\beta_2},$$

the shapes of both modes differ just slightly from the unperturbed shape $f(\beta_2, x)$; furthermore, the shifts for the two modes are unequal and are in opposite directions. The unperturbed shape can be determined with the aid of (9). We find

$$\frac{p_1}{p_2} = \tan\left[w_2p_2 + \frac{1}{2}\left(\tan^{-1}\frac{p_1}{p_2} - \tan^{-1}\frac{p_3}{p_2}\right)\right] \equiv \tan U$$

for even-numbered modes and

$$\frac{p_1}{p_2} = -\cot U$$

for odd-numbered modes. Thus,

$$\begin{aligned} f(\beta_2, x) &= \tan U \sin p_2x + \cos p_2x \\ &= \sec U \cos (p_2x - U) \\ &= \frac{(p_1^2 + p_2^2)^{\frac{1}{2}}}{p_2} \cos\left[p_2x - w_2p_2 - \frac{1}{2}\left(\tan^{-1}\frac{p_1}{p_2} - \tan^{-1}\frac{p_3}{p_2}\right)\right] \end{aligned} \tag{24}$$

for even-numbered modes and, similarly,

$$f(\beta_2, x) = \frac{(p_1^2 + p_2^2)^{\frac{1}{2}}}{p_2} \sin\left[p_2x - w_2p_2 - \frac{1}{2}\left(\tan^{-1}\frac{p_1}{p_2} - \tan^{-1}\frac{p_3}{p_2}\right)\right] \tag{25}$$

for odd-numbered modes. The mode shapes in guide IV can be determined in an analogous manner, with perturbations performed about β_4 instead of β_2. We leave the details to the reader.

The above results are not surprising. If the double waveguide system has a mode with propagation constant β_+ or β_- which is close to the propagation constants β_2 and β_4 of modes that can travel in the individual isolated waveguides, then we would indeed expect the shape of that double waveguide mode to deviate only slightly in each waveguide from the shape of the mode that could propagate in the isolated waveguide.

The amplitudes of $e_\pm(x)$ in guides II and IV prove to be more interesting. Let the arbitrary amplitude A in (3) be written as $A_\pm$ for $e_\pm(x)$, and let $B_\pm^{\mathrm{II}}$ and $B_\pm^{\mathrm{IV}}$ denote the amplitudes of the modes in guides II and IV, respectively. Then (3) and (24) or (25) show that the mode amplitudes in guide II are given (to our order of approximation) by

$$B_\pm^{\mathrm{II}} = A_\pm \frac{(p_1^2 + p_2^2)^{\frac{1}{2}}}{p_2}. \tag{26}$$

In guide IV, the amplitudes are, by (3),

$$B_{\pm}^{\mathrm{IV}} = A_{\pm}\left[C_2C_3\left(1 + \frac{p_1}{p_2}T_2\right)(1 - XT_3)\frac{(p_3^2 + p_4^2)^{\frac{1}{2}}}{p_4}\right]\Bigg|_{\beta_{\pm}}. \quad (27)$$

Care must be taken in the evaluation of $C_3(1 - XT_3)$ at $\beta_{\pm}$. Since $X(\beta_+) < 1$, $Y(\beta_+) < 1$, and by (8),

$$T_3 = \frac{X + Y}{1 + XY} = \tanh(\tanh^{-1} X + \tanh^{-1} Y),$$

we have

$$C_3 = \cosh(\tanh^{-1} X + \tanh^{-1} Y) = \frac{1 + XY}{(1 - X^2)^{\frac{1}{2}}(1 - Y^2)^{\frac{1}{2}}},$$

so that

$$C_3(1 - XT_3)|_{\beta_+} = \left(\frac{1 - X^2}{1 - Y^2}\right)^{\frac{1}{2}}\Bigg|_{\beta_+} \doteq \left[\frac{(2\Delta + \bar{\delta})X'}{\bar{\delta}Y'}\right]^{\frac{1}{2}}.$$

We find in a similar manner that

$$C_3(1 - XT_3)|_{\beta_-} = -\left[\frac{\bar{\delta}X'}{(2\Delta + \bar{\delta})Y'}\right]^{\frac{1}{2}}.$$

Thus the mode amplitudes in guide IV are

$$B_{+}^{\mathrm{IV}} = A_+C_2\left(1 + \frac{p_1}{p_2}T_2\right)\frac{(p_3^2 + p_4^2)^{\frac{1}{2}}}{p_4}\left[\frac{(2\Delta + \bar{\delta})X'}{\bar{\delta}Y'}\right]^{\frac{1}{2}}.$$

$$B_{-}^{\mathrm{IV}} = -A_-C_2\left(1 + \frac{p_1}{p_2}T_2\right)\frac{(p_3^2 + p_4^2)^{\frac{1}{2}}}{p_4}\left[\frac{\bar{\delta}X'}{(2\Delta + \bar{\delta})Y'}\right]^{\frac{1}{2}}. \quad (28)$$

We observe that the mode amplitudes will have the *same* signs in one waveguide and the *opposite* signs in the other. Thus, $e_+(x)$ might be termed quasi-even and $e_-(x)$ quasi-odd. More startling, however, is the realization that, if $\Delta > 0$, then the ratio $|B_+^{\mathrm{II}}/B_-^{\mathrm{II}}| = A_+/A_-$ may be quite different from the ratio $|B_+^{\mathrm{IV}}/B_-^{\mathrm{IV}}| = (A_+/A_-)[(2\Delta + \bar{\delta})/\bar{\delta}]$. As we see in the next section, this will have serious implications when we consider the double waveguide system as a device for transferring power.

For future reference, we write $e_{\pm}(x)$ for a system consisting of two symmetric, but not necessarily identical, waveguides ($K_1 = K_3 = K_5$). In this instance, (9) and (10) imply that

$$C_2[1 + (p_1/p_2)T_2] = 1,$$
$$C_4[1 + (p_1/p_4)T_4] = 1,$$

so that we have by (3) and our previous analysis

$$
\begin{aligned}
e_+(x) &= A_+ \exp p_1 x \qquad x < 0 \\
&= A_+ \frac{(p_1^2 + p_2^2)^{\frac{1}{2}}}{p_2} \cos p_2(x - w_2) \qquad 0 < x < 2w_2 \\
&= A_+[-X \sinh p_1(x - 2w_2) + \cosh p_1(x - 2w_2)]|_{\beta_+} \\
&\qquad\qquad 2w_2 < x < 2(w_2 + w_3) \\
&= A_+ \left[\frac{(2\Delta + \bar{\delta})}{\bar{\delta}} \frac{X'}{Y'} \right]^{\frac{1}{2}} \frac{(p_1^2 + p_4^2)^{\frac{1}{2}}}{p_4} \cos p_4(x - 2w_2 - 2w_3 - w_4) \\
&\qquad\qquad 2(w_2 + w_3) < x < 2(w_2 + w_3 + w_4) \\
&= A_+ \left[\frac{(2\Delta + \bar{\delta})}{\bar{\delta}} \frac{X'}{Y'} \right]^{\frac{1}{2}} \exp p_1(2w_2 + 2w_3 + 2w_4 - x) \\
&\qquad\qquad 2(w_2 + w_3 + w_4) < x,
\end{aligned} \tag{29}
$$

$$
\begin{aligned}
e_-(x) &= A_- \exp p_1 x \qquad x < 0 \\
&= A_- \frac{(p_1^2 + p_2^2)^{\frac{1}{2}}}{p_2} \cos p_2(x - w_2) \qquad 0 < x < 2w_2 \\
&= A_-[-X \sinh p_1(x - 2w_2) + \cosh p_1(x - 2w_2)]|_{\beta_-} \\
&\qquad\qquad 2w_2 < x < 2(w_2 + w_3) \\
&= -A_- \left[\frac{\bar{\delta} X'}{(2\Delta + \bar{\delta}) Y'} \right]^{\frac{1}{2}} \frac{(p_1^2 + p_4^2)^{\frac{1}{2}}}{p_4} \cos p_4(x - 2w_2 - 2w_3 - w_4) \\
&\qquad\qquad 2(w_2 + w_3) < x < 2(w_2 + w_3 + w_4) \\
&= -A_- \left[\frac{\bar{\delta} X'}{(2\Delta + \bar{\delta}) Y'} \right]^{\frac{1}{2}} \exp p_1(2w_2 + 2w_3 + 2w_4 - x) \\
&\qquad\qquad 2(w_2 + w_3 + w_4) < x.
\end{aligned} \tag{30}
$$

V. BEAT LENGTH AND POWER TRANSFER

Suppose the two modes $E_\pm = e_\pm(x) \exp i(\omega t - \beta_\pm z)$ travel down the double waveguide device. Since they travel at different phase velocities, the quasi-even and quasi-odd modes will alternately add and (partially) cancel in each waveguide. Hence, power is transferred between the two waveguides.

The beat length L over which this transfer takes place is given by

$$
L = \frac{\pi}{2(\bar{\delta} + \Delta)} = \frac{\pi}{2\delta[1 + (\Delta/\delta)^2]^{\frac{1}{2}}}. \tag{31}
$$

Note that, if the degenerate shift δ is fixed, then as the mismatching Δ increases, the beat length L decreases. We can conceive of ways to tune the double waveguide device and thus to change the beat length. This might be useful for light switching or modulation.

It is important to learn just how much power can be transferred in the waveguide system. Suppose, for definiteness, that we excite just one waveguide at $z = 0$ (say, guide IV), with the intent of transferring power to guide II via the mode-coupling mechanism. If guides II and IV have degenerate modes ($\Delta = 0$), then as the modes travel down the waveguide system, they will alternately add and then cancel (to order δ^2) in each waveguide, with addition occurring in one waveguide when at the same position cancellation occurs in the other. If guides II and IV have nondegenerate modes ($\Delta > 0$), however, then complete cancellation cannot take place in *both* waveguides: by (27) and (28), we see that if the amplitudes of $e_+(x)$ and $e_-(x)$ are adjusted so that the modes cancel at $z = 0$ in guide II, then they will never cancel fully in guide IV.

From a practical point of view, a parameter which is likely to be of interest in this matter is the fraction of the *total* power introduced into the system which can be transferred into guide II. If the modes are poorly confined, an appreciable fraction of the power carried by a waveguide may actually be outside the high dielectric guiding region. If the reader is interested in the fraction of the power which can be transferred not only to the guiding region of guide II, but also to its vicinity, we would need a power transfer ratio G to be defined by

$$G = \int_{-\infty}^{a} [e_+(x) + e_-(x)]^2 dx \Big/ \int_{-\infty}^{\infty} [e_+^2(x) + e_-^2(x)] dx,$$

where a is some number between $2w_2$ and $2(w_2 + w_3)$ which defines the "boundary" between guides II and IV. The numerator of this expression, then, is proportional to the power carried by the entire guide II.

Unfortunately, for a general asymmetric waveguide system, it is not at all clear how to define the position of the "boundary" between the two waveguides. If the system consists of two symmetric waveguides which are nearly identical (except for a small deviation if the modes are slightly mismatched), then it seems clear that the boundary should be midway between the two dielectric regions, i.e., at $a = 2w_2 + w_3$. By using (29) and (30), we find in this instance that we have to first order

$$G = [1 + (\Delta/\delta)^2]^{-1}.$$

Thus for perfectly matched waveguides ($\Delta = 0$), the power transfer is complete, to first order. As the mismatching increases, the power transfer ratio decreases rapidly.

Complete power transfer (to first order) is a direct consequence of assumption (17), which implies little overlap between the field associated with guide II and that associated with guide IV. A higher order perturbation analysis would show that in fact there is some field overlap and that, even if $\Delta = 0$, the power transfer is not complete. As the waveguide separation increases, there would be less field overlap and the power transfer would be more nearly complete.

For a general asymmetric waveguide system, we might define the "boundary" between the two waveguides to be, say, at the position where the "quasi-even" field attains its minimum. Such a definition can be cumbersome to apply mathematically. In general, though, we would expect results similar to those obtained for the symmetric system. If the modes are degenerate and one waveguide is excited, then virtually all the power can be transferred to the vicinity of the other waveguide. The power transfer ratio decreases as the mismatching increases.

It will be instructive to introduce a second power transfer ratio F, which can be defined precisely. It will be the fraction of the total power introduced into the system which can be transferred into the *high dielectric region* of guide II, the waveguide which was originally unexcited. If terms of order Δ/β_0 are neglected, this power transfer ratio is defined by

$$F = \int_0^{2w_2} [e_+(x) + e_-(x)]^2 dx \bigg/ \int_{-\infty}^{\infty} [e_+^2(x) + e_-^2(x)] dx, \qquad (32)$$

where the mode amplitudes A_+ and A_- are equal.

If the modes are poorly confined in guide II, the power transfer ratio F may be considerably less than unity even if the waveguides are perfectly matched ($\Delta = 0$). The definition of F is concerned only with the power which can be transferred into the high dielectric region of guide II; hence, F depends upon the confinement factor of the waveguide (to be defined below) as well as upon the amount of mismatching Δ.

Evaluation of (32) can be very messy for the general case of two asymmetric waveguides. We simplify the subsequent analysis and yet retain its essential flavor by assuming that the double waveguide structure is composed of two symmetric, but not necessarily identical, waveguides ($K_1 = K_3 = K_5$). The modes $e_\pm(x)$ are then given by (29) and (30). For the numerator of F, we find

$$\int_0^{2w_2} [e_+(x) + e_-(x)]^2 dx = 4A_+^2 \frac{(p_1^2 + p_2^2)}{p_2^2} \int_0^{2w_2} \cos^2 p_2(x - w_2) dx$$

$$= 4A_+^2 \frac{(p_1^2 + p_2^2)}{p_2^3} [p_2 w_2 + \tfrac{1}{2} \sin 2p_2 w_2].$$

But since (9) implies that

$$\sin 2p_2 w_2 = \frac{2p_1 p_2}{p_1^2 + p_2^2}$$

for a symmetric waveguide, we have

$$\int_0^{2w_2} [e_+(x) + e_-(x)]^2 dx = \frac{4A_+^2}{p_2} \left[p_2 w_2 \left(\frac{p_1^2 + p_2^2}{p_2^2} \right) + \frac{p_1}{p_2} \right].$$

Similar procedures can be used in the evaluation of the denominator of F. Special care must be taken in evaluating the integral in the interval $2w_2 < x < 2(w_2 + w_3)$, where $e_+(x)$ and $e_-(x)$ have the same functional description, but the function is evaluated at β_+ and β_-, respectively. In the resulting analysis, C_3 and S_3 must be evaluated at $\beta_\pm$. Procedures similar to those used following (27) are helpful. The power transfer ratio turns out to be

$$F = \frac{4(P_{\text{II}}/p_2)}{2[(P_{\text{II}}/p_2) + (1/p_1)] + 2[(P_{\text{IV}}/p_4) + (1/p_1)](1 + 2\Delta^2/\delta^2)X'/Y'},$$

where

$$P_{\text{II}} = p_2 w_2 \left(\frac{p_1^2 + p_2^2}{p_2^2} \right) + \frac{p_1}{p_2},$$

$$P_{\text{IV}} = p_4 w_4 \left(\frac{p_1^2 + p_4^2}{p_4^2} \right) + \frac{p_1}{p_4}. \tag{33}$$

The expression for F can be simplified. By using (19), (20), and (33), we find that

$$\frac{P_{\text{II}}}{p_2} + \frac{1}{p_1} = \frac{X'}{Y'} \left(\frac{P_{\text{IV}}}{p_4} + \frac{1}{p_1} \right), \tag{34}$$

so that

$$F = \frac{P_{\text{II}}/p_2}{[(P_{\text{II}}/p_2) + (1/p_1)][1 + (\Delta^2/\delta^2)]}. \tag{35}$$

Both (34) and (35) have interesting physical interpretations. In order to discuss them, we make a brief digression. Suppose that, instead of the double waveguide system, we just have an isolated guide II, which for generality we assume is not necessarily symmetric. (In the notation of Section II, we have $K_3 = K_4 = K_5$.) If the electric field is given by $e(x) \exp i(wt - \beta_2 z)$, then the power carried by the entire waveguide is proportional to

$$P = \int_{-\infty}^{\infty} e^2(x)dx$$

and the fraction of the total power which is confined to the high dielectric region is given by

$$C = \int_0^{2w_2} e^2(x)dx \bigg/ \int_{-\infty}^{\infty} e^2(x)dx.$$

It is not difficult to show that

$$P = \frac{p_1^2 + p_2^2}{2p_1p_2^2}\left[2p_1w_2 + 1 + (p_1/p_3)\right],$$

where $e(x)$ was assumed to have unit amplitude at $x = 0$. Upon comparing this expression with (19), we find that

$$X' = -\frac{2\beta_0}{p_3}\frac{(p_2^2 + p_3^2)}{(p_1^2 + p_2^2)}P,$$

so that X' is related in a simple manner to the power carried by the waveguide with which it is associated. Now if the isolated guide II happens to be symmetric, then it is also true that

$$P = \frac{P_{\mathrm{II}}}{p_2} + \frac{1}{p_1},$$

$$X' = -\frac{2\beta_0}{p_1}P.$$

Since a similar relationship holds for an isolated guide IV, (34) follows. The physical implication of (34) is that, if a single waveguide is excited in a matched double waveguide system ($\Delta = 0$), then the power is distributed between the two modes in such a manner that

$$\int_{-\infty}^{\infty} e_+^2(x)dx = \int_{-\infty}^{\infty} e_-^2(x)dx.$$

If an isolated guide II is symmetric, then the confinement factor C can be shown to be given by

$$C = \frac{P_{\mathrm{II}}/p_2}{(P_{\mathrm{II}}/p_2) + (1/p_1)}. \tag{36}$$

Hence, the power transfer ratio F is simply given by

$$F = C[1 + (\Delta/\delta)^2]^{-1}. \tag{37}$$

Note that F is completely independent of the parameters of guide IV, as well as of w_3 because of assumption (17).

We remark once more that these results are first-order approximations. If a higher order perturbation analysis were undertaken, it would show that F is also dependent upon the amount of field overlap between the two waveguides.

It is important to note that the amount of mismatching Δ can have a significant effect on the power transfer ratio: as Δ increases, F decreases rapidly. We might remark that, while the beat length L depends upon δ and the ratio Δ/δ, the power transfer ratio F depends only upon Δ/δ. Thus, by proper device design, it may be possible to adjust δ and Δ/δ to get both appreciable power transfer and a desirable beat length.

VI. A NUMERICAL EXAMPLE

To illustrate our results, let us give an example, using parameters which could be realized in a $GaAs-Al_xGa_{1-x}As$ heterostructure. We consider two waveguides which are each symmetric, but which have different widths and dielectric step heights. Our intent is to excite one waveguide and then to transfer power into the other by means of mode coupling. The parameters of one waveguide, say guide II, are taken to be fixed. The width of the other waveguide is considered a variable. For any given width $2w_4$, we adjust the dielectric height K_4 so that the propagation constants β_2 and β_4 for the zeroth order TE modes of the two waveguides match.

We learn how the degenerate shift δ varies with the spacing $2w_3$ between the two waveguides and with the width $2w_4$ of guide IV. We look at the beat length L and the power transfer ratio F for an idea of how much mismatching of the propagation constants β_2 and β_4 can be tolerated. We then discuss the amount of mismatching $2\Delta = |\beta_4 - \beta_2|$ which might occur in a practical situation and show how tuning can reduce this.

To be specific, suppose that

$$\begin{aligned} K_2 &= 11.868, \\ K_1 &= 0.9K_2 = 10.681, \\ w_2 &= 0.1\ \mu\text{m}, \\ k &= 5.4494 \times 10^4\ \text{cm}^{-1}. \end{aligned}$$

The dielectric constant K_2 corresponds to an index of refraction $K_2^{\frac{1}{2}} = 3.445$ of GaAs at a wavelength $\lambda = 2\pi\, k^{-1} = 1.153\ \mu\text{m}$. The halfwidth w_4 of guide IV will be assumed to vary between 0.1 μm and 0.8 μm.

For the zeroth order TE mode of a symmetric waveguide, we find that (9) reduces to

$$\begin{aligned} \tan p_2 w_2 &= p_1/p_2, \\ p_1 &= (\beta_2^2 - k^2 K_1)^{\frac{1}{2}}, \\ p_2 &= (k^2 K_2 - \beta_2^2)^{\frac{1}{2}}. \end{aligned} \tag{38}$$

The above parameters for guide II give us

$$\begin{aligned} \beta_2 &= 1.8049 \times 10^5\ \text{cm}^{-1}, \\ p_1 &= 2.9311 \times 10^4\ \text{cm}^{-1}, \\ p_2 &= 5.1631 \times 10^4\ \text{cm}^{-1}. \end{aligned}$$

Equation (38) holds for guide IV if each subscript 2 is replaced by a 4. For any given value of w_4, we adjust K_4 so that β_4 is the same as β_2. Some values of K_4 and p_4 are given in Table I.

The parameters for K_1, K_2, w_2 and any given pair K_4, w_4 then define two waveguides which have a degenerate mode at the given wavelength λ. The degenerate shift δ is given by (21) for symmetric waveguides:

$$\delta = \frac{p_1^2 p_2 p_4 (1 - \tanh w_3 p_1)}{2\beta_2[(p_1^2 + p_2^2)(p_1^2 + p_4^2)(1 + p_1 w_2)(1 + p_1 w_4)]^{\frac{1}{2}}}.$$

Figure 2 shows δ as a function of w_3 for various values of w_4. We see that the coupling decreases rapidly as the waveguide separation increases or as the width of guide IV increases. The beat length L, then, will increase rapidly with w_3 or w_4 if Δ is small enough. If, however, the

Table I — Values of K_4 (and hence p_4) needed to match β_4 for the zeroth order TE mode in guide IV (of halfwidth w_4) to the β_2 for the corresponding mode in guide II

w_4 (μm)	K_4	p_4 (10^4 cm^{-1})
0.1	11.868	5.1631
0.2	11.381	3.4917
0.3	11.222	2.7340
0.4	11.145	2.2763
0.5	11.100	1.9619
0.6	11.071	1.7295
0.7	11.051	1.5494
0.8	11.037	1.4048

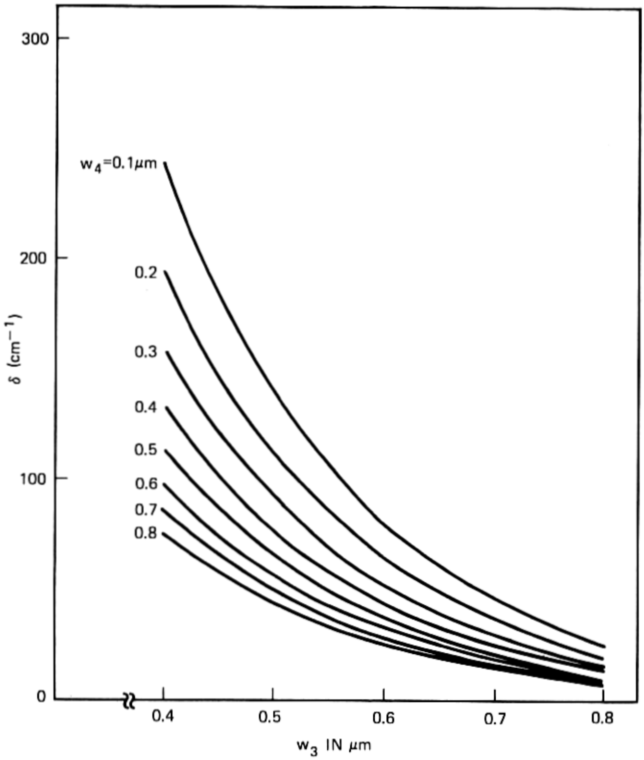

Fig. 2—The degenerate shift δ as a function of half the distance between the two waveguides w_3, with the halfwidth w_4 of guide IV as a parameter.

waveguides cannot be fabricated to match as closely as desired, the story can be different. If we take, quite arbitrarily, $\Delta = 100\ \text{cm}^{-1}$, we would find for example from (31) and the data in Fig. 2 that if $w_4 = 0.6\ \mu\text{m}$,

$$L = \begin{Bmatrix} 0.113 \\ 0.137 \\ 0.150 \\ 0.157 \end{Bmatrix} \text{mm} \quad \text{if} \quad w_3 = \begin{Bmatrix} 0.4 \\ 0.5 \\ 0.6 \\ \infty \end{Bmatrix} \mu\text{m} \qquad (\Delta = 100\ \text{cm}^{-1})$$

while if $\Delta = 0$,

$$L = \begin{Bmatrix} 0.162 \\ 0.279 \\ 0.490 \end{Bmatrix} \text{mm} \quad \text{if} \quad w_3 = \begin{Bmatrix} 0.4 \\ 0.5 \\ 0.6 \end{Bmatrix} \mu\text{m} \qquad (\Delta = 0).$$

Thus, L is reduced significantly and changes less rapidly with w_3 if Δ is large enough. Similar results are found if w_3 is fixed and w_4 varies.

Next, suppose we wish to excite one waveguide and to transfer power to the other one. If the propagation constants for the two waveguides were perfectly matched ($\Delta = 0$), then by (35) to (37) the power transfer ratio F would be the confinement factor C which is plotted in Fig. 3. The upper curve is used if guide II is excited and power is transferred to guide IV; the lower curve is used if guide IV is originally excited and power is transferred to guide II. The power transfer ratio is larger if power is transferred from a narrow guide II to a wide guide IV than if the power is being transferred the other way, since the power is more tightly confined within the guiding region in the wider waveguide.

Lest the reader become confused, we recall that F is defined as the fraction of the *total* power introduced into the system which can be transferred into the high dielectric region of the guide which was originally unexcited. F does *not* concern itself with how much power in the high dielectric region of one guide can be transferred into the high dielectric region of the other guide.

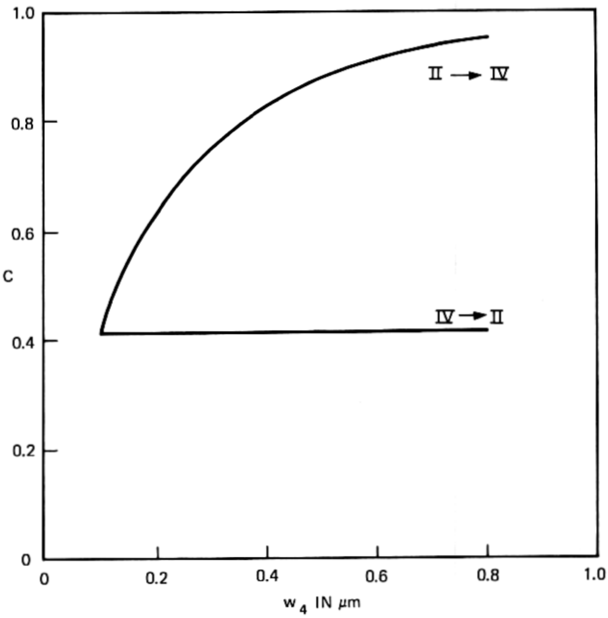

Fig. 3—The confinement factor C for degenerate modes. Guide II is fixed, and the halfwidth w_4 of guide IV varies.

Again, the mismatching Δ can have a significant effect. If, for example, we assume that $w_4 = 0.6\ \mu$m and transfer power from guide II to guide IV, we find from (35) (evaluated with the parameters for guide IV) that if $\Delta = 100\ \text{cm}^{-1}$,

$$F = \begin{Bmatrix} 0.66 \\ 0.44 \\ 0.22 \end{Bmatrix} \quad \text{if} \quad w_3 = \begin{Bmatrix} 0.4 \\ 0.5 \\ 0.6 \end{Bmatrix} \mu\text{m}.$$

(The maximum value of F is 0.91 for $\Delta = 0$.)

If we know that the fabrication procedure will likely make Δ of significant size, then the only way (for given waveguide parameters) to get good power transfer is to make δ large enough. A trade-off thus must be made between good power transfer and a long beat length.

Fortunately, tuning can be a viable alternative to making such a trade-off for badly matched waveguides. To tune a device which has already been fabricated, we would need to alter one or more slab widths or dielectric heights.

Some possible methods of tuning are to change K_2 or K_4 by altering the free carrier density or using the electro-optic effect, or to change the outer slab levels K_1 or K_5 by diffusion or ion implantation. This can be achieved, in principle, by growing at least one waveguide with a small gradient in the slab width. Phase matching then can be achieved by lateral positioning of the light beam, which travels approximately perpendicular to the gradient of the slab width.

We compute a possible value of Δ for a specific example and then see how much tuning is needed to reduce Δ to zero. Suppose that (for any given w_4) the double waveguide device is fabricated according to the specifications for matched propagation constants, but that there are slight errors in w_2, w_4, $K_2 - K_1$, and $K_4 - K_1$. Assume the following errors, which are probably reasonable if the device is fabricated by molecular beam epitaxy:[5] the *ratio* w_2/w_4 is nearly constant, and w_2 varies by $\pm 0.02\ \mu$m; the *ratio* $(K_2 - K_1)/(K_4 - K_1)$ is nearly constant, and $K_2 - K_1$ varies by ± 0.10. We shall take K_2 to be fixed. Then the extreme cases would be given by $w_2' = 0.12\ (0.08)\ \mu$m, $K_1' = 10.781\ (10.581)$, and

$$w_4' = (w_4/w_2)w_2',$$

$$K_4' = K_1' - \left(\frac{K_2 - K_1'}{K_2 - K_1}\right) K_1 + \left(\frac{K_2 - K_1'}{K_2 - K_1}\right) K_4,$$

where the primed parameters refer to the values in the fabricated

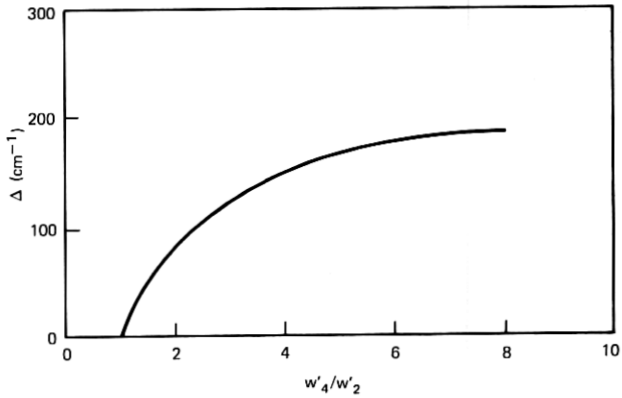

Fig. 4—The mismatching Δ as a function of the ratio of the guide widths.

device. We just treat the extreme case for which $w_2' = 0.12\ \mu$m and $K_1' = 10.781$, since the other gives changes of comparable size.

We find from (38) that, with these fabrication errors, $\beta_2 = 1.8156 \times 10^5$ cm^{-1} and β_4 varies from 1.8156×10^5 cm^{-1} for $w_4' = w_2'$ to 1.8119×10^5 cm^{-1} for $w_4' = 8w_2'$. The resulting values of $\Delta = \frac{1}{2}|\beta_4 - \beta_2|$ are plotted in Fig. 4. Although both β_2 and β_4 have changed significantly from the value 1.8049×10^5 cm^{-1} for which the device was designed, they both change in the same direction, so Δ reflects a less radical change.

In tuning the system, suppose we consider guide IV, and hence β_4, as fixed for any given w_4; we shall alter either the dielectric height or the symmetry of guide II to adjust β_2.

If we lower K_2 to make β_2 match β_4, we find by (38) that the altered values K_2' are those given in Table II. The change in K_2 is thus less than 0.4 percent. Such a change is feasible with free carrier injection

Table II — Values of K_2 needed to tune the mismatched waveguide system

w_4'/w_2'	K_2'	% change
1	11.868	0
2	11.846	0.19
3	11.840	0.24
4	11.831	0.31
5	11.829	0.33
6	11.825	0.36
7	11.824	0.37
8	11.823	0.38

Table III — Values of K_1 needed to tune the mismatched waveguide system

w_4'/w_2'	K_1'	% change
1	10.781	0
2	10.716	0.6
3	10.697	0.8
4	10.670	1.0
5	10.663	1.1
6	10.657	1.2
7	10.647	1.2
8	10.643	1.3

and is about an order of magnitude larger than can be handled completely by the electro-optic effect.

Another possible method of tuning would be to make guide II asymmetric. If we keep $K_2 = 11.868$, $K_3 = 10.781$, $w_2' = 0.12\ \mu\text{m}$, and alter K_1 to match β_2 to β_4, we use (9) in the form

$$p_1 = \frac{p_2 \tan 2w_2'p_2 - p_3}{1 + (p_3/p_2) \tan 2w_2'p_2}$$

to get p_1 and then use (4) to find K_1. The altered values K_1' are given in Table III. The change is thus no more than 1.3 percent. This can be handled by diffusion or ion implantation.

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